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# A Fortran II Computer Program for Calculation of Concentration-Dependent Sedimentation Coefficients\*

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#### Summary

A computer program for calculation of concentration-dependent sedimentation coefficients is presented. Provision is made for rejection of outlying experimental points. The method yields extremely rapid and precise results from original data without any intermediate steps.

Use of a computer program for calculation of concentration-dependent sedimentation constants assures a more rapid, precise, and sophisticated method. Excellent treatments of Fortran programming are given by Organick (1), Germain (2), and Stein and Munro (3). A general review of computation methods of ultracentrifugation was presented by Trautman (4) and processing of analytical ultracentrifugal data was given by Chun and Gehrke (5). In this discussion will be presented a detailed treatment of a complete Fortran program that can be utilized to get these sedimentation constants direct from the initial data without any intervening steps such as graph plotting or calculations. Rejection of outlying results is also a built-in feature. If desired, modification of the program will also allow for second moment calculations.

The sedimentation constant (or coefficient) is defined by the following differential equation:

$$S = \frac{1}{\omega^2 r} \frac{dr}{dt} = \frac{1}{\omega^2} \frac{d\ln r}{dt}$$

<sup>o</sup> This work was supported in part by U.S. Public Health Service Grant HE-05943 and Public Health Service Fellowship 5-F2-HE-29,038-02.

```
DIMENSION X(6)+AVERL(6+6)+T(6)+SLOPE(6)+CONC(6)
   DIMENSION SD(6) . REDA(6) . K(6) . SLAPE(6) . CA(6)
   DIMENSION A(6) + C(6) + CALCY(6+6) + DIF(6+6)
   DIMENSION VEBA(6)+DIFF(6)+DIFS(6+6)
   DIMENSION NSAVE (6) + MAO(6)
    DIMENSION COLCY(6+6)+DAF(6+6)+DAFS(6+6)
    DIMENSION SO(6) + CULCY(6) + DOF(6) + DOFS(6)
13 READ 1.(A(1).1=1.6)
    PUNCH 1+(A(1)+1=1+6)
    READ 2.NCONC
    DO 600 M=1.NCONC
    READ 2.NT
   NSAVE (M) = NT
    MA=NSAVE (M)
   DO 400 J=1.MA
   READ 2+NX
    READ 3. (X(I). I=1.NX)
    AVER=0.
    DO 300 I=1+NX
300 AVER=AVER+X(1)
    EXENX
400 AVERL(M+J)=LOG((AVER+0+48449/EX)+5+72)/2+3026
    READ 3.(T(1).1=1.NT)
    SUMX=0.
    SUMY=0.
    SUMXY=0.
    SUMXX≖U.
    DO 500 L=1.MA
    SUMX=SUMX+T(L)
    SUMY=SUMY+AVERL(M+L)
    SUMXX=SUMXX+T(L)**2
500 SUMXY=SUMXY+AVERL(M+L)*T(L)
    G=MA
    DENOM=SUMX**2-G*SUMXX
    SLOPE (M) = (SUMX*SUMY-G*SUMXY)/DENOM
    C(M)=(SUMX*SUMXY-SUMY*SUMXX)/DENOM
    DO 501 IM=1.MA
    CALCY(M+IM)=SLOPE(M)+T(IM)+C(M)
    DIF(M.IM)=AHS(CALCY(M.IM)-AVERL(M.IM))
501 DIFS(M.IM)=DIF(M.IM)**2
    SDS=0.
    DO 502 IN=1.MA
502 SDS=SDS+DIFS(M.IN)
    R=MA
    SD(M)=(SDS/(R-1+))**+5
    DIFF(M)=DIF(M+1)
    K(M) = 1
    DO 503 1P=2.MA
    REBA(M)=DIFF(M)-DIF(M.IP)
    IF (REBA(M))504.504.503
504 DIFF(M)=DIF(M+IP)
    K(M) = IP
503 CONTINUE
    VEBA(M)=D1FF(M)-1.5*SD(M)
    IF (VEBA(M))505+505+506
505 MOU=1
    MAO(M) = 1
    GO TO 510
506 MB=K(M)
```

FIG. 1.

```
PUNCH 4.M.T(MB) AVERL(M.MB)
    SAMX=0.
    SAMY=0.
    SAMXX=U.
    SAMXY=0.
    DO 509 IQ=1.MA
    1F(K(M)-10)508.509.508
SUB SAMX=SAMX+T(1Q)
    SAMY=SAMY+AVERL (M+1Q)
    SAMXX=SAMXX+T(IQ)*#2
    SAMXY=SAMXY+T(10) *AVERL(M .10)
509 CONTINUE
    F=MA-1
    DENAM=SAMX++2-F+SAMXX
    SLAPE(M)=(SAMX*SAMY-F*SAMXY)/DENAM
    CA(M) = (SAMX*SAMXY-SAMY*SAMXX)/DENAM
    DO 700 JA=1.MA
    COLCY(M,JA)=5LAPE(M)+T(JA)+CA(M)
    DAF (M.JA) = ABS (COLCY (M.JA) - AVERL (M.JA))
700 DAFS(M.JA)=DAF(M.JA) **2
    SO$=0.
    DO 702 J8=1.MA
    IF (K(M)-JB)701.702.701
701 SOS=SOS+DAFS(M.JH)
702 CONTINUE
    RA=MA-2
    SO(M)=(SOS/RA)**.5
    MOD=2
    MAO(M)=2
    PUNCH 5+SLAPE(M)+CA(M)
510 PUNCH 6.M. SLUPE (M) .C(M)
    IF(M00-2)512,511,511
511 SLOPE(M)=SLAPE(M)
    C(M) = CA(M)
512 CONTINUE
600 SLOPE(M)=1+/(SLOPE(M)*9+7943E-10)
    DO 601 J=1.NCONC
    PUNCH 7, J, SD(J)
    IF (MAO(J)-2)703+704+703
704 PUNCH 18+J+50(J)
703 PUNCH 8
    MC=NSAVE(J)
    DO 601 1=1.MC
    PUNCH11+T(1)+AVERL(J+1)+CALCY(J+1)+DIF(J+1)
601 CONTINUE
    READ 3. (CONC(1).1=1.NCONC)
    SMX=0.
    SMY=0.
    SMXY=0.
    SMXX=0.
    DO 610 LE=1.NCONC
    SMX=SMX+CONC(LB)
    SMY=SMY+SLOPE(LB)
    SMXX=SMXX+CONC(Lb)**2
610 SMXY=SMXY+CONC(Lb)*SLUPE(LB)
    H=NCONC
    DENM=SMX++2-H+SMXX
    SLOPE = (SMX*SMY-H*SMXY)/DENM
    CU=(SMX*SMXY-SMY*SMXX)/DENM
```

FIG. 1. (continued).

```
DO 611 LC=1.NCONC
    CALCY (LC+LC)=SLOPF*CONC (LC)+CU
    DIF(LC+LC)=ADS(CALCY(LC+LC)-SLUPE(LC))
611 DIFS(LC+LC)=DIF(LC+LC)**2
    500=0.
    DO 612 LD=1+NCONC
612 SOD=SOD+D1FS(LD+LD)
    R=NCONC
    SD(1)=(SOD/(R-1.))**5
    DIFF(1)=DIF(1+1)
    KA=1
    DO 614 LE=2.NCONC
    REBA(LE)=DIFF(1)=DIF(LE+LE)
    1F (REBA(LE))613.613.614
613 UIFF(1)=DIF(LE+LE)
    KA≈LE
614 CONTINUE
    VEBA(1)=DIFF(1)=1.3*SD(1)
    IF (VEBA(1))615+615+616
615 M00=1
    MAD=1
    GO TO 620
616 PUNCH 9.CONC(KA) .SLOPE(KA)
    SIMX=0.
    SIMY=0.
    SIMXX=U.
    SIMXY=0.
    DO 618 LG=1+NCONC
    IF (KA-LG)617+618+617
617 SIMX=SIMX+CONC(LG)
    SIMY=SIMY+SLOPE(LG)
    SIMXY=SIMXY+CONC(LG) #SLOPE(LG)
    SIMXX=SIMXX+CONC(LG) ##2
618 CONTINUE
    Z∓NCONC-1
    DENIM=SIMX**2-Z*SIMXX
    BOOB=(SIMX*SIMXY- SIMY*SIMXX)/DENIM
    E # 1 • / BOOB
    SLOM= (SIMX*SIMY-Z*SIMXY)/DENIM
    DO 750 JC=1.NCONC
    CULCY (JC) = SLOM*CONC (JC) + BOOB
    DOF(JC)=ABS(CULCY(JC)-SLOPE(JC))
750 UOFS(UC)=DOF(UC)**2
    SAS=C.
    DO 762 JD=1.NCONC
    IF (KA-JD) 761 . 762 . 761
761 SAS=SAS+DOFS(JU)
762 CONTINUE
    R0=NCONC-2
    SA= (SA5/RO) **+5
    MAD=2
    M00=2
620 D=1./CO
    PUNCH 12
    DO 630 LZ=1.NCONC
630 PUNCH 15+CONC(LZ)+SLOPE(LZ)+CALCY(LZ+LZ)+DIF(LZ+LZ)
   PUNCH 14.5D(1)
    IF (MAD-2) 770 . 771 . 770
771 PUNCH 19.5A
```

FIG. 1. (continued).

```
770 CONTINUE
    PUNCH 10.D
    1F(MO0-2)801.800.801
BUU PUNCH 16.E
801 CONTINUE
   PUNCH 17
  FORMAT(1UA4)
   FORMAT(115)
2
  FORMAT(7F10.2)
з
  FORMAT(5HCONC++12+10HREJ+ PT T=+F10+6+2X+6HAVERL=+F10+6)
   FORMAT(16HCORRECTED SLOPE=+E20+6+5X+10HINTERCEPT=+E20+6)
5
   FORMAT(BHIN CONC++12+6HSLUPE=+E20+6+5X+10HINTERCEPT=+E20+6)
6
   FORMAT(/BHIN CONC++12+16HTHE STAND+ DEV+=+F10+6)
7
                   т
                         +12H
                                   AVERL +12H
                                                    CALCY .
   FORMAT(12H
8
            DIF
   112H
    FORMAT(//8HIN CONC., E15.6.2X.14HREJ. 1/5 APP.=. 15.6)
Q
10 FORMAT(//27HTHE SEDIMENTATION CONSTANT=+E20+6)
    FORMAT(4F12+6)
11
                                       1/5 APP .
12 FORMAT(7/15H
                    CONC
                               •13H
   114H CALC 1/S APP.5X 4H DIF)
14 FORMAT(28HTHE STAND. DEV. OF 1/S APP.=. E12.6)
15 FORMAT(4E14+6)
16 FORMAT(22HCORRECTED SED. CONST.=.E12.6)
17 FORMAT(9HABCDEFGH1)
18 FORMAT(BHIN CONC++12+18HCORR+ STAND+ DEV+=+F10+6)
19 FORMAT(JUHSTAND+ DEV+ OF CORR+ 1/5 APP+=+E20+0)
20 FORMAT(F5+2)
    GO TO 13
    END
```

FIG. 1. Program.

### Program notes

- 1. It is possible for an investigator if he so desires to raise or lower the standard deviation factor in statements 503 + 001, and 614 + 001.
- 2. In statement 400, the magnification factor, 0.48449, can be changed to fit the ultracentrifuge in use.
- 3. In statement 600, the factor  $9.7943 \times 10^{-10}$  can be altered if the speed of the rotor is changed.

### Definition of input symbols

**NCONC** = number of concentrations

- T = the time at a particular interval
- NT = number of times
  - $\mathbf{X} =$ value of micrometer reading from reference line to the maximum ordinate
- NX = number of x values

#### Summary of arrangement of input data cards

- 1. Title of experiment (columns 1-24)
- 2. Number of concentrations (fifth column)
- 3. Number of times for first concentration (fifth column)
- 4. Number of x values at first time and first concentration (fifth column)
- 5. x values at first time and first concentration

#### Continued at bottom of p. 1133

SAM	PLE A10			
6 5				
3				
1.9452	1.9429	1.9428		
1.9910	1.9915	1.9915	1.9920	
2.0330	2.0337	2.0340		
3 2.0819	2.0827	2.0827		
4	2.1315	2 13.74	2 1210	
0.	d.	16.	24.	32.
5				
1.9413	1.9353	1.9378	1.9355	
1.9900	1.9872	1.9866	1.9851	
2.0355	2.0349	2.0333		
2.0866	2.0864			
2.1383	2.1382			
0.	8.	16.	24.	32.
5				
1.9391	1.9390			
1.9806	1.9804			
2.0272	2.0284	2.0250	2.0272	2.0276
2.0796	2.0775	2.0782	2.0780	
2.1318	2.1296	2.1317		
J.	8.	16.	24.	32.
3				
1.9371	1.9390	1.9380		
1.9950	1.9920	1.9925	1.9953	
2.0525	2.0527			
2.1038	2.1028	2.1040		
2.1570	2.1573			
0.	8.	16.	24.	32.
5				
1.9262	1.9270	1.9293	1.9300	
1.9813	1.9810	1.9823		
2.0389	2.0373	2.0360	2.0377	
-				

FIG. 2.

2.0987	2.0977	2.0972			
3 2.1520	2.1500	2.1535	24.	32.	
5				220	
3					
1.9895	1.9915	1.9915			
3					
2.0520	2.0530	2.0528			
2					
2.1080	2.1080				
3					
2.1653	2.1637	2.1633			
3					
2.2299	2.2324	2.2312			
0.	8.	16.	24.	32.	
1.000000	.750000	.666667	.500000	.333333	.25000000

FIG. 2. Example of input data.

where r = distance of boundary from center of rotation,  $\omega$  = angular velocity in radians per second, and t = time in seconds.

Therefore, if  $\ln r$  is plotted versus time, the slope divided by  $\omega^2$  gives the sedimentation constant directly. Also, if desired, this can be corrected to the standard state by multiplying by a correction factor (6).

If the polymer is pure and has a narrow molecular weight dis-

col. 1	10	11	20	21	30	31	40	41	50
Numbe	r of <b>x</b> val	ues at sec	ond ti	me and	first cone	entratio	1		
col. 1	10	11	20	21	30	31	40	41	50
Do sam	e for all :	<b>r</b> values c	f first	concent	ration				
6. Time va	alues at f	irst conce	ntratio	n					
col. 1	10	11	20	21	30	31	40	41	50
Repeat	cards 3–6	6 for all co	oncent	rations					
7. Concen	tration va	alues							
col. 1	10 1	1 20	21	30	31	40 41		50 51	60
Notes									

- 1. If a particular time is omitted, this fact must be accounted for in card 3 and its value omitted in the proper order in card 6.
- 2. If a particular concentration is omitted, this fact must be accounted for in card 2 and its value omitted in the proper order in card.

3. Cards must be checked and rechecked for proper content and order.

SAMPLE A1	0			
CONC. IREJ. PT T	= 16+000000	AVERL=	·826409	
CORRECTED SLOPE=	18=248	828E-05	INTERCEPT=	82.358957E-02
IN CONC. 1SLOPE =	18.249	062E-05	INTERCEPT =	82.356950E-02
CONC. 2REJ. PT T	= 16.000000	AVERL=	.826440	
CORRECTED SLOPE	19+638	281E-05	INTERCEPT=	82-337917E-02
IN CONC. 25LOPE=	19-636	437E-05	INTERCEPT=	82-336303E-02
IN CONC. 38LOPE=	18.99	1255-05	INTERCEPT=	82-330578E-02
CONC. ARE L. RT T	- 16-000000		- 937005	32.000.00-02
	21 47	1715.05	INTERCERT-	D3 3420315 03
IN CONC. ASLOPES	21 474	51712-08	INTERCEPT-	92 34E016E-02
IN CONC. 45LOPE=	21.470		INTERCEPT-	52+345015E=02
IN CONC. SELOPE	22.10	5750E-05	INTERCEPT=	82.304/25E-02
IN CONC. BSLOPE	23.100	00002-05	INTERCEPTE	82.50/8466-02
IN CONC. ITHE ST	AND. DEV.=	•000049		
IN CONC. 1COPR.	STAND. DEV.=	•00002	5	
Ŧ	AVERL	CALCY	DIF	
1000000	823577	823569	80000e	
B+000000	825075	825029	•00045	
16.000000	<b>.</b> 826409	826489	•000080	
24.000000	827939	827949	•000009	
32.000000	829444	·829409	•0000 <b>35</b>	
IN CONC. 2THE ST	AND. DEV.=	.000039		
IN CONC. 2CORR.	STAND. DEV.=	+00001	9	
T	AVERL	CALCY	DIF	
0-00000	.823383	.823363	-000020	
8-000000	.824953	.824034	-000019	
16-000000	-826440	.826505	+000064	
20.000000	.828047	- 828074	-000008	
22-000000	020001	820647	000030	
32.000000	027001	102964/	+000034	
IN CONC. STHE ST		.000110		
TN CONC. STHE ST	ANU/0 I'C.V.0-	•000110	DIE	
		CALCT		
0.0000000	+BZ3433	•823305	•000127	
8.000000	•824740	•824817	♦ PP0076	
16.00000	.826205	•H26329	•con123	
24.000000	·827811	•B27841	•000029	
32+000000	•B29456	829353	•000103	
IN CONC. 4THE ST	AND. DEV.=	■000070		
IN CONC. ACORP.	STAND. DEV.≠	•00002	4	
т	AVERL	CALCY	DIF	
0.000000	·823400	<ul> <li>B23450</li> </ul>	•000049	
8.000000	825156	·825168	•000011	
16+000000	·827005	·826886	.000119	
24+00000	.828599	P28604	•0000C5	
32+000000	830269	.830322	.000052	
IN CONC. STHE ST	AND. DEV.=	+000053		
Ŧ	AVERL	CALCY	DIF	
0.000000	.823087	.823047	.00040	
8.000000	.824773	.824815	+000042	
16.000000	+826531	.826583	.000052	
24.00000	A828422	829352	.000070	
32.000000	•B30104	.830120	.000015	
	1000104		•••••	
IN CONC. ATHE ST	AND. DEV	.000094		
THE COME DOUGLE OF				

FIG. 3.

T	AVERL	CALCY	DIF	
0.000000	·825066	.825076	.000010	
8.000000	·827005	.826924	.00081	
16.000000	•828738	·828772	.000033	
24.000000	.830486	•B30620	·CO0134	
32.000000	·832565	<b>.</b> 832468	•00097	
IN CONC. 66.	666700E-02 R	FJ. 1/5 APP.	= 54•0 <b>266</b> 31E•	+11
CONC	1/S APP	CALC 1/S	APP DIF	
10.000000E-0	1 55-948910E+	11 56+665074	E+11 71.616400	1E+09
75+000000E-0	2 51+990397E+1	11 52+655798	E+11 66+540100	18+09
66+666700E-0	2 54-026631F+1	11 51+319378	E+11 27.072530	E+10
50.000000E+0	2 47.541157E+:	11 48+646523	F+11 11+053660	1E+10
33+333300E-0	2 46+191349E+1	11 45.973667	E+11 21.768200	1E+09
25.00000CE-0	2 44+199221E+1	11 44+637247	E+11 43+802600	1E+09
THE STAND. DE	V. OF 1/S APP.	= +139613E+	12	
STAND. DEV. 0	F CORR. 1/S AF	P.= .	13539656E+11	
THE SEDIMENTA	TION CONSTANT:	24.6	13583F-14	

CORRECTED SED. CONST. = .246976E-12

FIG. 3. Example of calculation.

Notes for calculation

```
1. Abbreviations:
```

- a. **REJ.** PT = rejected point
- b. CORR. STAND. DEV. = standard deviation of new line after rejection of outlying point
- c. Rej. 1/s app. = rejected  $1/s_{APPARENT}$
- d. STAND. DEV. OF CORR. 1/S APP. = standard deviation of new line after rejection of outlying 1/SAPPARENT
- e. CORRECTED SED. CONST. = corrected sedimentation constant after rejection of outlying 1/SAPPARENT

**f.** CALCY = calculated AVERL

- 2. Since the intermediary as well as final results are printed out, it is possible for the investigator to spot check calculations if he so desires by plotting his own curves and reject his own outlying points and compare.
- 3. By comparing the corrected and uncorrected standard deviations one can see whether there is justification in rejecting an outlying point.
- 4. In difference column, computer truncates in print-out last digit rather than rounding it off.
- 5. The sedimentation constant reported should be rounded off to three significant figures.

tribution, the schlieren pattern obtained will be closely symmetrical and it will be possible to determine the maximum ordinate by bisecting the schlieren fringes without resorting to second moment calculations.







#### Notes for Flow Chart

- X = micrometer reading on comparator of distance from outer edge of inner reference hole to boundary
- AVERL = natural logarithm/2.303 (average of X value at a particular time and conc.
   × magnification factor<sup>o</sup> + distance from outer edge of inner reference hole to center of rotation<sup>†</sup>)

3. 
$$\frac{1}{\mathbf{S}_{app.}} = \frac{60(\omega^2)}{2.303(d \ln x/dx)}$$
$$= \frac{(2\pi \times \text{speed of rotor in rpm}1/60)^2}{\text{slope} \times 2.303/60} = \frac{1}{\text{slope } (9.7943 \times 10^{-10})}$$

#### **EXPERIMENTAL**

#### Materials

Commercial heparin sodium, derived from beef lungs, was obtained from Organon, Inc. Heparin is a charged polysaccharide. For use, 2% solutions of heparin in 0.5 M NaCl solution were dialyzed for 48 hr with constant magnetic stirring and under refrigeration against 0.5 M NaCl solution.

- \* Magnification factor of our ultracentrifuge is 0.48449.
- † This distance is 5.72 cm, taking into consideration the stretching of the rotor.
- ‡ Speed used is 59,780 rpm.

#### Methods

All sedimentation velocity runs were made using the Beckman-Spinco Model E analytical ultracentrifuge. The An-D rotor was used with the R.T.I.C. system set at 20°C with a speed of 59,780 rpm. Single-sector capillary-type synthetic boundary cells with Epon centerpieces were filled with 0.25 ml of solute and about 0.2 ml of solvent in the reservoir. Precision volume measurements were made with Hamilton microliter syringes equipped with Chaney adaptors. The dialysate was used as solvent for all dilutions. Photographs were taken at 8-min intervals on Kodak II-G spectroscopic plates. Cells were checked for leakage in each run.

Computer used was the IBM 1620-(1) with 40K memory, equipped with Fortran or Kingston Fortran compiler. Calculations were checked by plotting and calculator.

## DISCUSSION OF PROGRAM

Two to five readings are made on a photographic plate from the reference line obtained from the inner reference hole of the counterbalance to a line bisecting the schlieren fringes. An average of these readings is calculated, multiplied by the magnification factor of the ultracentrifuge, and added to the distance of reference line from the center of rotation. This is done at each particular time interval. The logarithms of these values are plotted versus the particular times, from which the slopes, intercepts, and equation of the lines are obtained. If there is an outlying point in any concentration, it is rejected by using a modification of the 2.5-D rule (7). Because of the precision of the ultracentrifuge it was found experimentally that a point should be rejected if the absolute difference between its calculated and actual value is greater than 1.5 times the standard deviations of the line. If a point has been rejected, the line is replotted omitting this point and a new slope, intercept, and standard deviation obtained. The above procedure is repeated for all the concentrations and the apparent sedimentation constants are calculated from the slopes. Then the reciprocals of the apparent sedimentation constants are plotted versus concentrations. Again an outlying point will be rejected, but this time if the absolute difference between the calculated and actual value is greater than 1.3 times the standard deviation of the line. This result was arrived at by repeated runs. If a point is rejected, the line is

replotted omitting this point and a new slope, intercept, and standard deviation obtained. Through extrapolation to zero concentration, the sedimentation constant is found both for the case where a point is rejected and where none is rejected. At least five concentrations should be used for this determination. (See Figs. 1 to 3.)

## CONCLUSIONS

Concentration-dependent sedimentation constants can be calculated on a computer. A program in Fortran which can be compiled either in Fortran or Kingston Fortran was presented. Results are obtained quickly and efficiently from initial data without intermediate plotting and calculation. Also a built-in mechanism is supplied for eliminating outlying data. The program can be easily modified to allow for second moment calculations and for use with the IBM 360 computer.

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### Zusammenfassung

Eine Rechenprogram zur Berechnung von konzentrationsabhängigen Sedimentationkoefiizienten wird angegeben. Ausserhalb liegende experimentelle Daten werden hierbei eliminiert. Die Methode ergibt rasche und genaue Ergebnisse direkt aus den ursprünglichen Daten ohne Zwischenstufen.

## Résumé

On présente un programme d'ordinateur pour calculer les coëfficients de sédimentation, dépendant de la concentration. On fait des arrangements pour rejetter les points expérimentaux à l'écart. La méthode donne des résultats extrêmement rapides et précises à partir des données originales sans procedés intermédiaires.

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